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Robust Battery Fuel Gauge Algorithm Development, Part 3: State of Charge Tracking

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Abstract—In this paper, we present a novel SOC tracking algorithm for Li-ion batteries. The proposed approach employs a *voltage drop model* that avoid the need for modeling the hysteresis effect in the battery. Our proposed model results in a novel reduced order (single state) filtering for SOC tracking where no additional variables need to be tracked regardless of the level of complexity of the battery equivalent model. We identify the presence of correlated noise that has been so far ignored in the literature and use this for improved SOC tracking. The proposed approach performs within 1% or better SOC tracking accuracy based on both simulated as well as HIL evaluations.

Index Terms—Battery management system (BMS), Battery fuel gauge (BFG), state of charge (SOC), online system identification, adaptive nonlinear filtering, extended Kalman filter (EKF), reduced order filtering.

I. INTRODUCTION

The battery SOC can be estimated through two distinct approaches: The first method, termed *Coulomb counting*, assumes knowledge of the initial state of charge and perfect knowledge of battery capacity to compute the remaining state of charge after accounting for the amount of Coulombs transferred from/into the battery. This approach has the following sources of error:

(A) Uncertainty in the knowledge of initial SOC;
(B) Uncertainty in the knowledge of battery capacity;
(C) Errors in measured Coulombs as a result of errors in the measured current and errors in time differences due to timing oscillator inaccuracies/drifts.

The second method of SOC estimation exploits the unique and stable relationship between the open circuit voltage (OCV) and SOC of the battery and allows one to compute the SOC for a measured OCV. However, it is possible to directly measure the OCV only when the battery is at rest. When the battery is in use, the dynamic relationship between the battery voltage and current has to be accounted for through parameter and state estimation approaches. The OCV-SOC based state of charge estimation approach has the following sources of error:

(a) Uncertainty in the modeling and parameter estimation of the dynamic electrical equivalent model of the battery
(b) Errors in measured voltage and current.

In [25]–[27], Plett reported several battery equivalent models and an extended Kalman filter (EKF) based approach to SOC tracking. Later, the same models were used in [28], [29]

that reported an unscented Kalman filter (UKF) filter based approach to SOC tracking. Similar EKF/UKF based SOC tracking approaches with varying enhancements were reported in [11], [12], [14], [15], [18]–[22], [30]. Some other notable approaches for SOC tracking are presented in [10], [16], [17].

In this paper, based on the normalized OCV modeling approach of [24], robust battery equivalent circuit model (ECM) parameter estimation approach of [6] and robust online battery capacity estimation approach of [7] we develop a robust SOC tracking algorithm. Other aspects of battery fuel gauge algorithm development, OCV modeling, equivalent circuit model (ECM) parameter estimation and battery capacity estimation, can be found in the same conference proceedings in [24], [6] and [7], respectively. The robustness of the proposed algorithm is rooted in the *normalized OCV modeling* approach presented in [24]; robustness in all other components of the BFG, parameter estimation [6], capacity estimation [7] and SOC tracking (present paper) are all based on normalized OCV modeling which yields unchanged OCV-SOC curve over temperature changes and battery aging. Extended details and other related aspects of the robust BFG algorithm will appear in [1]–[5], [23].

The rest of the paper is organized as follows. The system model and a formal statement of the battery SOC tracking problem is provided in Section II. Section III details the proposed online SOC tracking algorithm. The performance of the proposed SOC tracking approach is demonstrated through experimental results in Section IV, and the paper is concluded in Section V.

II. SYSTEM MODEL AND PROBLEM STATEMENT

The battery equivalent circuit model considered in this paper is shown in Figure 1. The terminal voltage $v[k]$ in terms of the components of the battery equivalent circuit is given by

$$v[k] = V_0(s[k]) + i[k]R_0 + i_1[k]R_1 + i_2[k]R_2 + h[k] \quad (1)$$

where $V_0(s[k])$ represents the open circuit voltage (in volts) of the battery at time k which is written here as a function of SOC at time k, $s[k] \in [0,1]$; $h[k]$ accounts for the hysteresis in the battery voltage; $i_1[k]$ and $i_2[k]$ are the currents through R_1 and R_2, respectively.

There are several nonlinear representations that approximate the OCV as a function of SOC. For example, the well known *combined model* is given in [25]. In this paper, we adopt a novel inverse polynomial-log-linear model for the
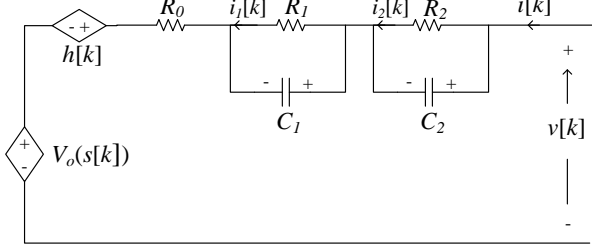
Fig. 1. **Equivalent circuit of a battery.** When the battery is at rest, $V_0(s[k])$ is the OCV of the battery. The OCV uniquely depends on the SOC of the battery, $s[k]$. When the battery is active, i.e., when there is current activity, the behavior of the battery is represented through the *dynamic equivalent circuit* consisting of a hysteresis component $h[k]$, a series resistance R_0 and two parallel RC circuits in series, (R_1, C_1) and (R_2, C_2). The discrete time is indicated using $[k]$.

representation of OCV in terms of SOC:

$$V_0(s[k]) = K_0 + \frac{K_1}{s[k]} + \frac{K_2}{s^2[k]} + \frac{K_3}{s^3[k]} + \frac{K_4}{s^4[k]} + K_5 s[k]$$
$$+ K_6 \ln(s[k]) + K_7 \ln(1 - s[k]) \quad (2)$$

where $K_0, K_1, K_2, K_3, K_4, K_5, K_6$ and K_7 can be estimated offline through OCV-SOC characterization.

The instantaneous change in SOC can be written in the following form (the subscript of x is introduced to indicate a state component)

$$x_s[k+1] \triangleq s[k+1] = s[k] + c_h \Delta i[k] \quad (3)$$

where $i[k]$ is in Amps,

$$c_h = \eta/3600 C_{\text{batt}} \quad (4)$$

is the Coulomb counting coefficient in $\text{Amp}^{-1}\text{sec}^{-1}$ (the factor 3600 indicates C_{batt} is in Ah (the commonly used unit for capacity) rather than Coulombs), C_{batt} is the capacity of the battery in Ampere hours (Ah), Δ is the sampling interval in seconds and η is a constant that depends on whether the battery is being charged or discharged, i.e.,

$$\eta = \begin{cases} \eta_c & i[k] > 0 \\ \eta_d & i[k] < 0 \end{cases} \quad (5)$$

It must be noted that (3) yields the instantaneous SOC of the battery. This way of computing the SOC is known in the literature as *Coulomb counting*. We also refer to this as the "predicted SOC" in this paper.

The current $i[k]$ is measured and the current measurement is subject to errors. The measured current $z_i[k]$ is written as

$$z_i[k] = i[k] + n_i[k] \quad (6)$$

where $n_i[k]$ is the current measurement noise, which is considered white, zero-mean and with known standard deviation (s.d.) σ_i.

The state equation (3) can be rewritten by substituting $z_i[k]$ for $i[k]$ according to (6) as follows

$$x_s[k+1] = x_s[k] + c_h \Delta z_i[k] - c_h \Delta n_i[k] \quad (7)$$

The currents through the resistors R_1 and R_2 can be written in the following form

$$x_{i_1}[k+1] \triangleq i_1[k+1] = \alpha_1 i_1[k] + (1 - \alpha_1)i[k] \quad (8)$$
$$x_{i_2}[k+1] \triangleq i_2[k+1] = \alpha_2 i_2[k] + (1 - \alpha_2)i[k] \quad (9)$$

where

$$\alpha_1 = e^{-\frac{\Delta}{R_1 C_1}} \quad (10)$$
$$\alpha_2 = e^{-\frac{\Delta}{R_2 C_2}} \quad (11)$$

By substituting the measured current $z_i[k]$ for $i[k]$, the currents in (8) and (9) can be rewritten as follows

$$x_{i_1}[k+1] = \alpha_1 x_{i_1}[k] + (1 - \alpha_1)z_i[k] - (1 - \alpha_1)n_i[k] \quad (12)$$

$$x_{i_2}[k+1] = \alpha_2 x_{i_2}[k] + (1 - \alpha_2)z_i[k] - (1 - \alpha_2)n_i[k] \quad (13)$$

The hysteresis voltage $h[k]$ is a nonlinear function of the load current and SOC of the battery [25]. The hysteresis process can be written as

$$x_h[k] \triangleq h[k] = f_h(x_s[k], i[k])x_h[k] + n_h[k] \quad (14)$$

where $n_h[k]$ is the process noise of the hysteresis model which is assumed white, zero-mean Gaussian and with s.d. σ_h. In this paper, we model hysteresis as the error in the predicted OCV of the BFG. More details can be found in Section III.

The voltage in (1) is a measured quantity and the measured voltage $z_v[k]$ is subject to errors. The measured voltage is written as

$$z_v[k] = v[k] + n_v[k]$$
$$= V_0(s[k]) + i[k]R_0 + i_1[k]R_1 + i_2[k]R_2$$
$$+ h[k] + n_v[k] \quad (15)$$

where $n_v[k]$ is assumed to be white Gaussian noise with zero mean and s.d. σ_v.

Now, by substituting (6), (8), (9) and (14), in (15), the following measurement model is derived

$$z_v[k] = V_0(x_s[k]) + z_i[k]R_0 + x_{i_1}[k]R_1 + x_{i_2}[k]R_2$$
$$+ x_h[k] + n_{z_v}[k] \quad (16)$$

where

$$n_{z_v}[k] = n_v[k] - R_0 n_i[k] \quad (17)$$

Now, given the instantaneous voltage and current measurements, $z_v[k]$ and $z_i[k]$, the objective of BFG is to track the instantaneous SOC of the battery $x_s[k]$. The presence of "nuisance" variables $x_{i_1}[k]$, $x_{i_2}[k]$, and $x_h[k]$ in the observation model (16) makes it a joint estimation problem, i.e., SOC and these variables have to be jointly estimated.

In addition, the following model parameters need to be estimated through system identification techniques:

- Battery capacity: C_{batt}

- Open circuit voltage model parameters: $K_0, K_1, K_2, K_3, K_4, K_5, K_6, K_7$
- Dynamic equivalent circuit model parameters: R_0, R_1, C_1, R_2, C_2
- Charging and discharging efficiencies: η_c, η_d
- Process noise variances: σ_i^2, σ_h^2
- Measurement noise variances: σ_v^2, σ_i^2

The requirement to estimate all the model parameters makes the SOC tracking problem more challenging. Further, the chemical properties of the battery change as a result of temperature changes, aging, and usage patterns and as a result these model parameters are subject to change over time. Hence the model parameters have to be re-estimated over time.

By exploiting the expectation maximization (EM) algorithm [13], [31], the authors developed a strategy in [8] for SOC tracking by jointly estimating many of the model parameters listed above. The EM approach involves the iteration of parameter estimation and SOC tracking over a large data window which results in significant computational and memory requirements. This is not desirable in portable mobile applications where the power requirement and compactness of the BFG is a major concern.

In this paper we assume that the OCV parameters $K_0, K_1, \ldots, K_7$ of the battery are estimated offline. The voltage and current measurement error standard deviations, σ_v and σ_i, respectively, are assumed to be available from the measurement circuitry design. The charging and discharging efficiencies, η_c and η_d, respectively, are assumed to be known through calibration. Hence, our objective in this paper is to develop an online SOC tracking algorithm by assuming the knowledge of battery capacity C_{batt} and the electrical equivalent model parameters R_0, R_1, R_2, C_1 and C_2 of the battery.

III. SOC TRACKING APPROACH

The objective of reduced order filtering is to track $x_s[k]$ while avoiding the need to track the redundant variables $x_{i_1}[k], x_{i_2}[k]$ and $x_h[k]$. First, let us rewrite (7) in the following form:

$$x_s[k+1] = x_s[k] + c_h \Delta z_i[k] + w_s[k] \tag{18}$$

where

$$w_s[k] = -c_h \Delta n_i[k] \tag{19}$$

is the process noise which is white with s.d.

$$\sigma_s = c_h \Delta \sigma_i \tag{20}$$

Now, let us rewrite the voltage measurement (16) as

$$z_v[k] = V_0(x_s[k]) + \mathbf{a}[k]^T \mathbf{b} + n_D[k] \tag{21}$$

where

$$\mathbf{a}[k]^T = \begin{bmatrix} v_D[k-1] & v_D[k-2] & z_i[k] & z_i[k-1] \\ -z_i[k-2] & 1 \end{bmatrix} \tag{22}$$

and the *voltage drop* is given by

$$v_D[k] = z_v[k] - V_0(x_s[k]) \tag{23}$$

where $\mathbf{b}$ is a parameter vector to be estimated and $n_D[k]$ is the measurement noise.

It must be noted that $\mathbf{a}[k]$ in (22) is defined in terms of the voltage drops $v_D[k-1]$ and $v_D[k-2]$. The estimated parameters derived in terms of the parameters of the battery equivalent model in Figure 1 as

$$
\begin{aligned}
\mathbf{b}(1) &\triangleq \alpha(k) = \alpha_1 + \alpha_2 \\
\mathbf{b}(2) &\triangleq \beta(k) = \alpha_1 \alpha_2 \\
\mathbf{b}(3) &= R_0 \\
\mathbf{b}(4) &\triangleq \check{R}_1 = (\alpha_1 + \alpha_2)R_0 - (1 - \alpha_1)R_1 - (1 - \alpha_2)R_2 \\
\mathbf{b}(5) &\triangleq \check{R}_2 = \alpha_1 \alpha_2 R_0 - \alpha_2(1 - \alpha_1)R_1 - \alpha_1(1 - \alpha_2)R_2, \\
\mathbf{b}(6) &\triangleq \hat{h}[k] = x_h[k] - \alpha(k)x_h[k-1] + \beta(k)x_h[k-2]
\end{aligned}
\tag{24}
$$

The measurement noise $n_D[k]$ is zero mean and with autocorrelation $\mathcal{R}_{n_D}(l)$.

Next, we discuss the significance of the voltage drop in estimating the parameters $\mathbf{b}$ in (21). Using (21), the voltage drop (23) can be written as

$$v_D[k] = \mathbf{a}[k]^T \mathbf{b} + n_D[k] \tag{25}$$

Given the voltage drop observations, the above model (25) can be used to linearly estimate $\mathbf{b}$. However, in order to get the voltage drop as an observation, one needs the knowledge of the SOC $x_s[k]$ for which a predicted value of SOC, $\hat{x}_s[k|k-1]$ or the updated SOC estimate $\hat{x}_s[k|k]$ can be used, i.e.,

$$v_D[k|k] = z_v[k] - V_0(\hat{x}_s[k|k]) \tag{26}$$

Later, we discuss how the predicted SOC $\hat{x}_s[k|k-1]$ and updated SOC $\hat{x}_s[k|k]$ are obtained (see (31) and (38), respectively.)

Let us now discuss the advantage of the voltage drop model. All the existing approaches for BFG use the voltage and current observations, $z_v[k]$ and $z_i[k]$, for model identification and SOC tracking.

Consider the conventional voltage observation model (16). The terms in $x_h[k]$ denote the hysteresis voltage which, as shown in (14), is a function of current $i[k]$, SOC $x_s[k]$ and time k. For example, when a battery experiences a load of 1A (which is heavy in mobile applications) for a few seconds, the magnitude of the resulting hysteresis is small compared to when the load was 1A continuously for $30\,\text{min}$. In addition, the magnitude of the hysteresis is a function of the SOC at that time as well.

When one uses the voltage observation (16) across battery terminals for model identification, the hysteresis $x_h[k]$ also needs to be modeled and the model parameters have to be estimated. A model for hysteresis in terms of SOC, current and time is nonlinear and not yet fully understood.

Another disadvantage of trying to model and estimate hysteresis is that it makes online model identification almost

impossible: Since hysteresis is a function of SOC, model identification requires the data spanning the entire range of SOC. This might be impossible at times since some applications might never use the battery from full to empty. Since hysteresis is also a function of current, model identification requires usage data spanning the possible load currents applied for various durations. Hence, complete modeling and model identification of hysteresis becomes impractical.

It is also important to note that estimating model parameters offline using sample batteries and then using those parameters in fuel gauging may not be satisfactory; some of the battery parameters are known to change based on usage patterns.

In this paper, we entirely avoid hysteresis modeling by introducing the voltage drop model described above. The voltage drop $v_D[k]$ represents the voltage across the internal battery model components R_0, R_1, R_2 and $x_h[k]$. The term $x_h[k]$ is purposely introduced in order to account for the errors in predicted SOC $\hat{x}_s[k|k-1]$ that is used to derive the voltage drop "measurement" $\hat{v}_D[k]$. One could think of $x_h[k]$ as an "instantaneous hysteresis" which must be corrected to zero by adjusting SOC estimate $\hat{x}_s[k|k]$.
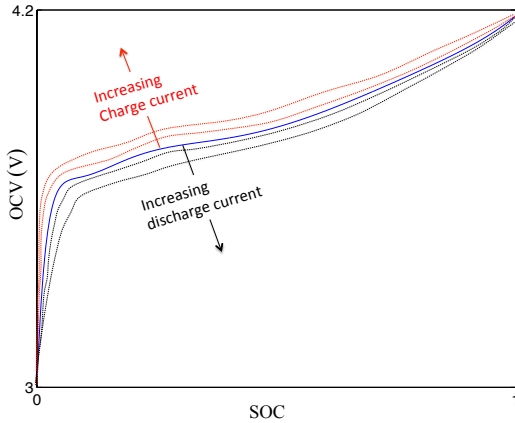

Fig. 2. **Generic description of hysteresis in Li-ion battery.** The figure above is used to explain that for a certain OCV (rested battery terminal), there are many possible values of SOC – depending on the path (in terms of load current and previous SOC). The true OCV-SOC relationship is unique, which lies in the middle and is shown in blue.

The OCV-SOC model (2) represents the *true OCV-SOC relationship* shown in solid line in Figure 2. Assuming that $\hat{x}_s[k|k] = x_s[k|k]$ in the voltage drop (23), the estimated value of $\hat{h}[k]$ in (24) will be $\hat{h}[k] = 0$. However, $\hat{h}[k] \neq 0$ implies errors in SOC estimates that is used in computing the voltage drop observation $v_D[k]$. Hence, the SOC tracking algorithm needs to adjust $\hat{x}_s[k|k]$ accordingly. We do this by employing the following modified observation model in place of (21)

$$z_v[k] = V_0(x_s[k]) + \tilde{\mathbf{a}}[k]^T \tilde{\mathbf{b}} + n_D[k] \qquad (27)$$

where

$$\tilde{\mathbf{a}}[k]^T = \begin{bmatrix} v_D[k-1] & v_D[k-2] & z_i[k] & z_i[k-1] \end{bmatrix}$$
$$-z_i[k-2] \end{bmatrix} \qquad (28)$$

$$\tilde{\mathbf{b}} = \begin{bmatrix} \alpha & \beta & R_0 & \check{R}_1 & \check{R}_2 \end{bmatrix}^T \qquad (29)$$

are obtained by removing the last element in $\mathbf{a}[k]^T$ and $\mathbf{b}$, respectively, i.e., without the hysteresis term. Later, we describe the significance of this modified observation model.

Furthermore, the following covariance is found between the process noise $w_s[k]$ in (18) and the measurement noise $n_D[k]$ in (27):

$$E\left\{w_s[k]n_D[k]\right\} \triangleq U[k] = R_0 c_h \Delta \sigma_i^2 \qquad (30)$$

Given the estimate of state of charge, $\hat{x}_s[k|k]$, and the associated variance $P_s[k|k]$, the following EKF recursion uses the voltage and current measurements $z_v[k+1], z_i[k], z_i[k+1]$ to yield the updated SOC estimate of $\hat{x}_s[k+1|k+1]$, and its associated variance $P_s[k+1|k+1]$. These steps also make sure that the SOC estimates are best adjusted to account for the covariance of (30) according to [9], Sec.8.3. The filtering recursions consist of the following:

$$\hat{x}_s[k+1|k] = \hat{x}_s[k|k] + c_h[k]\Delta z_i[k] \qquad (31)$$
$$P_s[k+1|k] = P_s[k|k] + \sigma_s^2 \qquad (32)$$
$$H[k+1] = \left.\frac{dz_v[k]}{dx_s[k]}\right|_{\hat{x}_s[k+1|k]} \qquad (33)$$
$$\hat{z}_v[k+1] = V_0(\hat{x}_s[k+1|k]) + \tilde{\mathbf{a}}[k]^T \hat{\tilde{\mathbf{b}}}[k] \qquad (34)$$
$$S[k+1] = H[k+1]P[k|k]H[k+1]^T \qquad (35)$$
$$+\mathcal{R}_{n_D}(0) + 2H[k+1]U[k] \qquad (36)$$
$$G[k+1] = \frac{P[k+1|k]H[k+1]^T + U[k]}{S[k+1]} \qquad (37)$$
$$\hat{x}_s[k+1|k+1] = \hat{x}_s[k+1|k]$$
$$+G[k+1]\Big(z_v[k+1] - \hat{z}_v[k+1]\Big) \qquad (38)$$
$$P_s[k+1|k+1] = (1 - G[k+1]H[k+1])P_s[k+1|k]$$
$$(1 - G[k+1]H[k+1])^T$$
$$+G[k+1]^2 \mathcal{R}_{n_D}(0) \qquad (39)$$

where $c_h[k]$ and $\hat{\mathbf{b}}[k]$ are the most recent estimates of the Coulomb counting coefficient and the model parameter vector, respectively. The effects of autocorrelation terms $\mathcal{R}_{n_D}(1)$ and $\mathcal{R}_{n_D}(2)$ in the measurement noise are ignored in the SOC tracking because it will require the estimation of a larger state and this is left for the future.

Now, let us briefly discuss the importance of using $\tilde{\mathbf{a}}[k]^T$ and $\tilde{\mathbf{b}}$ in the state-space model (18)–(27) for SOC tracking. Hysteresis can be thought of as an error in the OCV-SOC characteristic curve (see Figure 2). It is quite difficult to model and accurately estimate the hysteresis because it varies with the previous current and SOC (see (14)). However, the true OCV-SOC relationship can be easily estimated. Indeed, the

$V_0(x_s[k])$ in (27) is based on the true OCV-SOC model. Let us assume that the estimated hysteresis is $\hat{h}[k] = 10\,\mathrm{mV}$. This means the "perceived OCV" by the filter is $10\,\mathrm{mV}$ away from the actual OCV of the battery. For the BFG algorithm, the perceived OCV, $V_0(x_s[k])$, is directly (and monotonically) related to the SOC estimate $\hat{x}_s[k|k]$. In other words, if the perceived OCV of the filter is different from the actual OCV, so is the filter estimate $\hat{x}_s[k|k]$ from the true SOC of the battery. Hence, when the filter sees a drop of $10\,\mathrm{mV}$ in its predicted terminal voltage $\hat{z}_v[k+1]$ in (34), it adjusts its SOC estimate $\hat{x}_s[k+1|k+1]$ in (38) such that the "perceived OCV error" (or estimated hysteresis H) is (gradually) adjusted to zero. Hence, a good indication of proper functioning of the proposed approach is the estimated $\hat{h}[k]$ being always close to zero. This summarizes our novel hysteresis modeling approach of this paper where we do not actually try to model the hysteresis in terms of its current and SOC dependent paths, but rather, we force the OCV error "hysteresis" to be zero through a combination of (i) true OCV modeling, (ii) modeling for voltage drop, and (iii) filtering.

IV. SIMULATION RESULTS

In this section we provide performance analysis of the proposed BFG. First, we assess the performance using simulated battery data. Then, we use real data collected through hardware in the loop (HIL) experiments for performance assessment.

Typical battery usage data was simulated using the dynamic equivalent model which consists of a series resistance, the hysteresis component and a single RC circuit. The dynamic model parameters, assumed constant over the entire simulation, are selected as follows: $R_0 = 0.2\,\Omega$ $R_1 = 0.1\,\Omega$, $C_1 = 50\,\mathrm{F}$, $h[0] = 0\,\mathrm{V}$. It is assumed that the data is sampled at a constant $\Delta = 0.1$ seconds. Hence, the online model parameters $\mathbf{b} = \begin{bmatrix} \alpha & R_0 & \tilde{R}_1 & H \end{bmatrix}^T$ can be written as $\alpha = 0.9802$ (see (10)), $R_0 = 0.2$, $\tilde{R}_1 = 0.1941$, $H = h[k] - \alpha_1 h[k-1]$ where $h[k]$ is simulated as a constant bias. It is assumed that the true battery capacity is $C_\mathrm{batt} = 1.5\,\mathrm{Ah}$.

The OCV parameters are obtained by performing an OCV test on a Samsung battery (with serial number EB575152). This resulted in the following values for the OCV parameters: $K_0 = -3.0927$, $K_1 = 43.3102$, $K_2 = -7.4126$, $K_3 = 0.7908$, $K_4 = -0.0359$, $K_5 = -30.3610$, $K_6 = 59.0316$, $K_7 = -0.3932$.

the battery usage profile $i[k]$ was created in terms of load current as shown in Figure 3 (top). A random white noise with s.d. $\sigma_i = 10^{-3}\mathrm{A}$ was added to $i[k]$ (see (6)) in order to simulate the measured current $z_i[k]$. The current through R_1 is simulated for reference (shown in the second plot from the top of Figure 3). The true, noiseless, SOC is computed by Coulomb counting (18) by assuming that $x_s[0] = 1$ and assuming the knowledge of $i[k]$, i.e., by making $z_i[k] = i[k]$ and $w_s[k] = 0$ in (18); the true value of SOC, $x_s[k]$, is then computed. Now, assuming that the initial SOC, $x_s[0] = 1$, the measured battery terminal voltage was simulated by making use of the state-space model (18), (27) with the voltage measurement noise standard deviation $\sigma_v = 10^{-3}$ V. In Figure

3, we show these simulated quantities: true current profile $i[k]$, the current $x_1[k]$ through R_1, true SOC $x_s[k]$ and the (simulated) noisy battery terminal voltage $z_v[k]$, respectively, are shown from top to bottom in Figure 3. The simulated voltage $z_v[k]$ and current information $z_i[k]$ are fed to the BFG and the estimated parameters, capacity and SOC outputs of the BFG were recorded for analysis.

Next, we analyze the SOC tracking performances of the algorithm. First, we define the SOC tracking error as

$$\epsilon_{x_s}[k] = 100\sqrt{\frac{1}{M}\sum_{m=1}^{M}\left(x_s[k] - \hat{x}_{s,m}[k]\right)^2} \qquad (40)$$

where $\epsilon_{x_s}[k]$ is in percentage and $\hat{x}_{s,m}[k]$ is the estimated SOC at time k of the mth Monte-Carlo run.

Figure 4 shows the estimated SOCs as well as the SOC tracking errors. We also use this figure to demonstrate our novel state space model and the linear parameter estimation strategy for SOC tracking: First, we demonstrate SOC tracking based on model parameters estimated through conventional, unweighted LS method explained. The plots corresponding to this are labeled as LS1. Then, we demonstrate SOC tracking based on the new, weighted LS parameter estimation which is more accurate. The plots corresponding to this second approach are labeled as LS2.

V. CONCLUSIONS

This paper presented a novel state of charge tracking approach for Li-ion batteries. The proposed approach employs linear approach for parameter estimation and a robust online method for capacity estimation.

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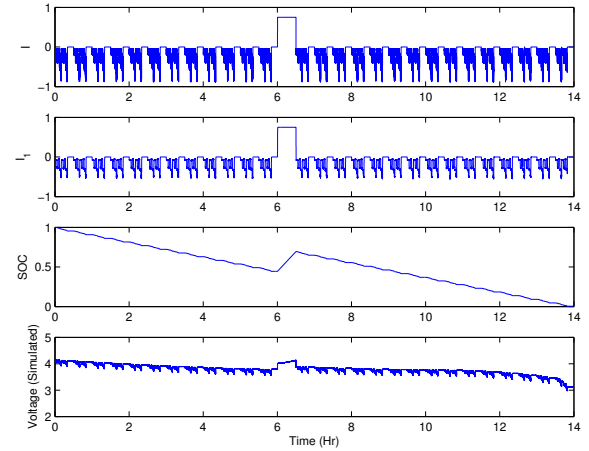
Fig. 3. **Validation data.** From top to bottom: Simulated load profile $i[k]$, computed current $x_1[k]$ through R_1, true SOC $x_s[k]$ and noisy battery terminal voltage $z_v[k]$.
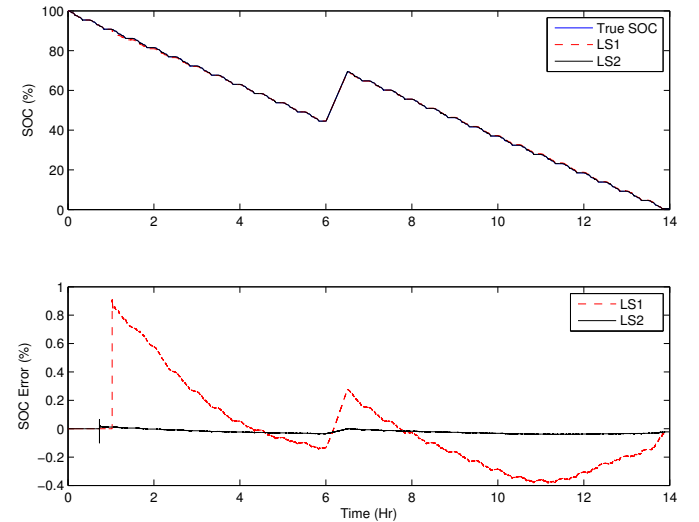

Fig. 4. **SOC tracking performance.**